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ABSTRACT

This manual presents a taxonomy for the classification of elementary school mathematics content and provides training in the use of that taxonomy. The taxonomy is designed to classify the content of various curricular materials as well as the content of actual classroom instruction. Three factors are used to define the taxonomy: (1) general intent; (2) nature of materials; and (3) operation. There are 6 levels of general intent, 13 levels of material, and 15 different operations. The manual elaborates on each factor by specifying the definition of the levels of each factor, conventions used in conjunction with each factor to eliminate certain ambiquities that might arise, and examples for each level. (MP)

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Research Series No. 4

A TAXONOMY FOR CLASSIFYING ELEMENTARY SCHOOL MATHEMATICS CONTENT

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revision of an earlier Taxonomy Training Manual

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Purpose

The purpose of this manual is to present a taxonomy for the classification of elementary school mathematics content and to provide training in the use of that taxonomy. The taxonomy is designed to classify the content of various curricular materials including tests, workbooks, textbooks, handouts, and worksheets as well as to classify the content of actual classroom instruction. In some respects the taxonomy may appear to be unrealistically detailed, while in other respects it may appear to gloss over important distinctions. An attempt was made to strike a balance between these two extremes, employing as the guiding principle that classification should be made at a level sufficient to inform curricular decisions.

Development of the taxonomy began with four independent, unstructured content analyses of the items on the Stanford Achievement Test (SAT), grade four. These analyses were pooled to form a first version of the taxonomy. Through an iterative process of applying the taxonomy to the SAT and then revising the taxonomy when the need was apparent the first taxonomy manual was written (Schmidt, et al.)* This manual was used to train raters who classified the content of four major standardized tests for grades 2 through 5. Further revisions were then made on the basis of this experience, and the need to accomodate the differing format of other instructional materials to be analyzed. This manual incorporates those revisions. Although a deliberate

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*Schmidt, W. H., Porter, A. C., Floden, R. E., and Freeman, D. J. Training manual for classification of the content of fourth grade mathematics (Research Series No. 4).

attempt has been made to provide an adequate description of the content of the entire elementary school mathematics curriculum, most of our work has concentrated on the fourth grade level. The taxonomy is, therefore, apt to function most effectively at or near this level.

Taxonomy

Three factors are used to define the taxonomy: (1) general intent, (2) nature of materials, and (3) operation. There are 6 levels of general intent, 13 levels of material, and 15 different operations. Conceptually, the taxonomy is formed by the intersection of the three factors, which results in 1170 possible cells for classification.

Use of the Manual

The remainder of this manual elaborates on each factor by specifying: (1) the definition of the levels of each factor, (2) conventions used in conjunction with each factor to eliminate certain ambiguities that might arise, and (3) examples for each level. Each example is followed by an explanation of the particular level that is illustrated. The classification for all three factors is then given in parenthesis. This complete analyses may not be meaningful until the entire manual has been read.

A summary of the taxonomy is provided after the description. Two sets of exercises are provided. In the first, comparatively simple discriminations are required to classify a given item on all three factors. The second exercise is more difficult in that it requires more subtle discriminators and is deliberately designed to refine the definition of each factor.

Taxonomy

Factor I: General Intent

1) Conceptual understanding with pictorial models (p.4)
2) Conceptual understanding without pictorial models (p.7)
3) Skill in reading graphs, tables, and measurement instruments (p.10)
4) Computation/numeration skills (p.12)
5) Applications involving graphs, tables, measurement instruments (p.14)
6) Applications without graphs, tables, etc. (p.16)

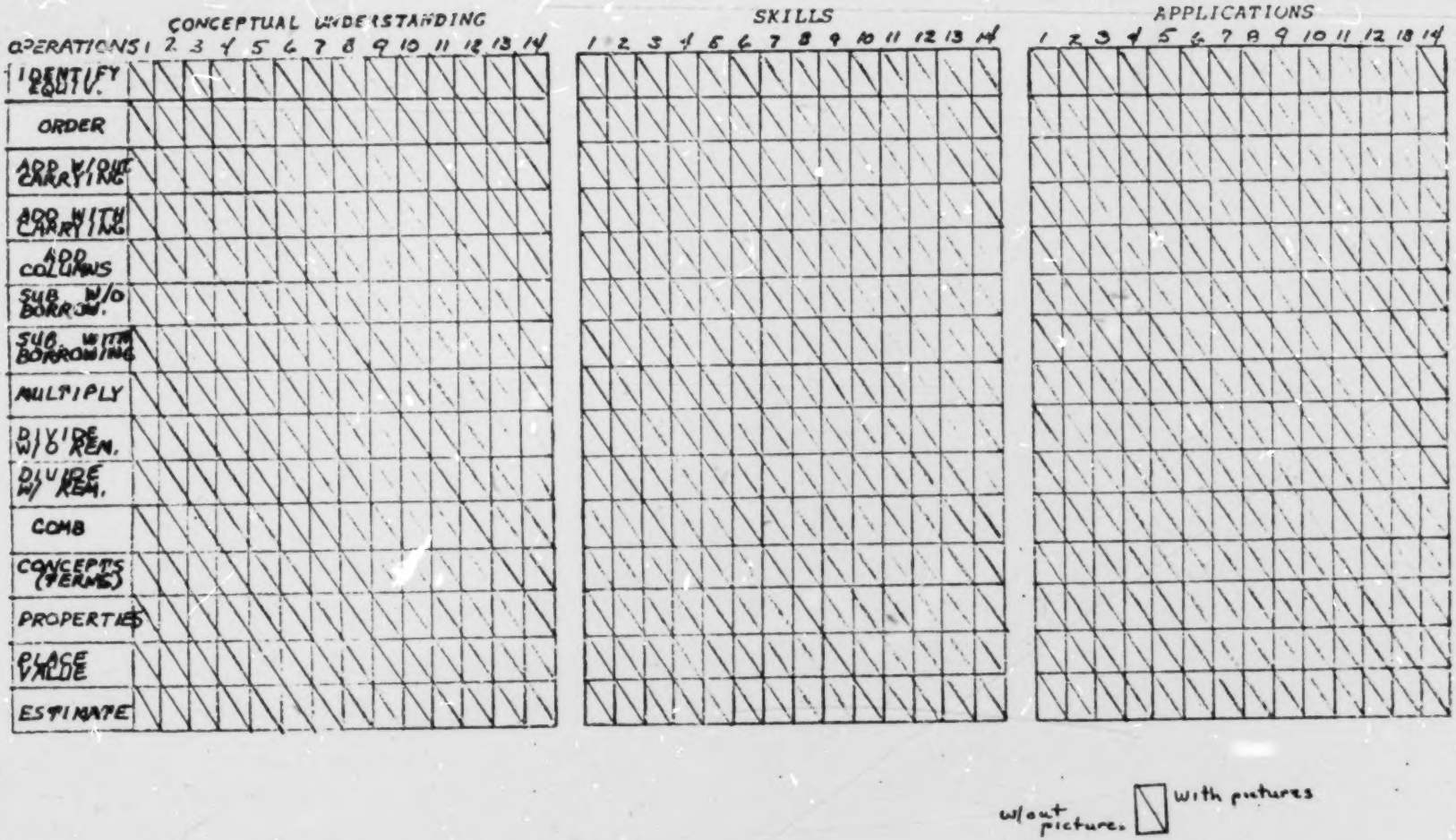
Factor II: Nature of Material (p.17)

1) Single digits or basic no. facts (p.35)
2) Single & multiple digit numbers (p.34)
3) Multiple digit numbers (p.33)
4) Number sentences/phrases (p.32)
5) Algebraic sentences/phrases (p.29)
6) Single or like fractions (p.28)
7) Unlike fractions (p.27)
8) Mixed numbers (p.26)
9) Decimals (p.25)
10) Percents (p.24)
11) Measurement (p.23)
12) Essential units of measurement (p.21)
13) Geometry (p.18)
14) Other (p.35)

Factor III: Operations

1) Identify equivalents (p.50)
2) Order (p.49)
3) Add without carrying (p.48)
4) Add with carrying (p.48)
5) Column addition (p.48)
6) Subtract without borrowing (p.47)
7) Subtract with borrowing (p.46)
8) Multiply (p.45)
9) Divide without remainder (p.45)
10) Divide with remainder (p.44)
11) Combination (p.43)
12) Apply concepts (terms) (p.40)
13) Apply properties (p.40)
14) Identify place value (p.38)
15) Estimate (p.37)

FIGURE 1 - TAXONOMY MATRIX



NATURE OF MATERIAL

1. sing. dig./basic facts
2. sing. & mult. digit
3. multiple digit
4. no. sen./phrase
5. alg. sen./phrase
6. sing./like frac.
7. unlike frac.
8. mixed no.
9. decimals
10. percents
11. measurement
12. essn. units of measurement
13. geometry
14. other

Factor I - General Intent

The six levels of Factor I describe both the general intent and the format in which it is presented. Three categories of intent are considered: conceptual understanding, skill acquisition, and the application of skills or concepts. For conceptual understanding, a distinction is made between units presented with pictorial models and those which are not illustrated by pictures. For mathematical skills a distinction is made between those which deal with the reading of graphs, tables and measurement instruments and those which deal with other mathematical skills. Two levels of applications are characterized by the presentation of the numerical information either in graphs, tables, etc. or in the context of a verbal problem. The resulting six levels are illustrated in figure 1 and defined in the following section.

Figure 1

Conceptual Understanding	- with pictorial models
	- without pictorial models
Mathematical Skills	- reading graphs, tables, etc.
	- computation/numeration skills
Applications	- involving graphs, tables, etc.
	- without graphs, tables, etc.

Levels 1 and 2 - Conceptual Understanding

If the task presented is intended to develop or test understanding of a particular idea or procedure used in mathematics, rather than to develop or test for proficiency in its use, it will be classified as conceptual understanding at Factor I. There are five broad categories of concept attainment

which are typically stressed in elementary school mathematics. These are (1) number (e.g., how many is "3"), (2) numeration (e.g., place value and fractions), (3) operations (e.g., what is addition), (4) geometry (e.g., parallel lines), and (5) measurement.

> Convention:
>
> Place value problems (see page 38) will be classified at one of the levels of conceptual understanding. Tasks which focus on the understanding of a specific term are classified at one of the levels of conceptual understanding when described by rule 1 page 40. If these terms fit rule 2 they will be classified as applications (levels 5 or 6) when the student must complete the calculation, and as conceptual understanding when no calculation is required.

A. Level 1 - Conceptual Understanding with Pictorial Models

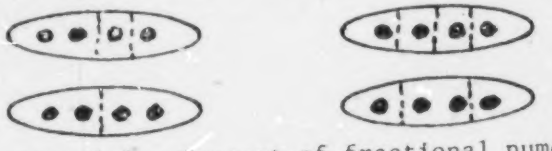
If the concept is illustrated using a pictorial model (e.g., pictures or number lines) or concrete objects (e.g., cuisenaire rods), it will be classified at Level 1 of Factor I.

Example 1. Which of the sets below are equivalent?

1) C and D
2) A and D
3) A and C
4) A and B

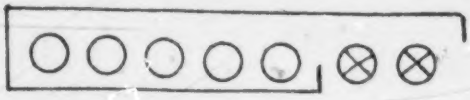
Classification: Level 1 - Concept of equivalent number . (1,1,1)

Example 2. Which of the sets below is separated into fourths?



Classification: Level 1 - Concept of fractional numeration. (1,6,1)

Example 3. Which of the facts below is best shown by the picture?



a) 7 + 2 = 9

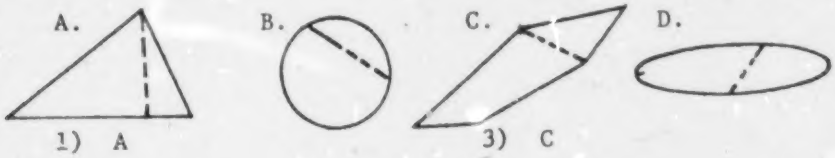
b) 4 + 3 = 7

c) 7 - 2 = 5

d) 7 - 3 = 4

Classification: Level 1 - Concept of subtraction (1,4,6)

Example 4. In which figure below is the dotted line a diagonal?



1) A

2) B

3) C

4) D

Classification: Level - Concept of a "diagonal". (1,13,12)

Example 5. The area of this figure is _____.

(Unit of Area is ☐.)

a) 7 units

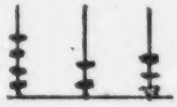
b) 12 units

c) 3 units

d) 48 units

Classification: Level 1 - Concept of area. (1,11,12)

Example 6. This value is _____.



a) 234

b) 423

c) 432

d) 324

Classification: Level 1 - Concept of place value. (1,3,14)

B. Level 2 - Conceptual Understanding Without Pictorial Models

If the intent of the task is conceptual understanding but pictorial models are not presented it will be classified at Level 2 of Factor I. Tasks which should be classified at this level include:

(1) problems that require some conclusions about the calculation to be performed rather than actual performance of the operation to find an answer

Example 7. How should you think of 5 ft. 4 in. in working this subtraction example?

a) 4 ft. 8 in.

b) 4 ft. 14 in.

c) 4 ft. 16 in.

d) 4 ft. 40 in.

$$\begin{array}{r} 5 \text{ ft. } 4 \text{ in.} \\ -\ 2 \text{ ft. } 7 \text{ in.} \\ \hline \end{array}$$

Classification: Level 2 - the question deals with understanding that 5 ft. 4 in. must be changed to 4 ft. 16 in. to solve the given problem. (2,12,1)

Example 8. How can the fraction 9/12 be reduced to simplest form?

a) Add 3 to the numerator and the denominator

b) Multiply the numerator and the denominator by 2

c) Divide the numerator and the denominator by 3

d) Divide the numerator by 3 and the denominator by 4

Classification: Level 2 - the student does not have to reduce the fraction, just understand the procedure. (2,7,1)

(2) problems in which a student must specify the appropriate operation or form a number sentence rather than provide the numerical answer to a calculation

Example 9. Paul had 50¢. He spent 15¢ for milk and found 5¢. What can you do to find out how much money Paul had then?

a. divide and add

b. subtract and add

c. divide and multiply

d. subtract and multiply

Classification: Level 2 - the student must understand which operations are to be used but not apply their use. (2,4,11)

(3) problems which require the student to identify information needed to solve a story problem

Example 10. If Sue wants to buy 3 notebooks for school, how much money does she need? To answer this question you need to know:

a) the size of the notebooks

b) the cost of each notebook

Classification: Level 2 - students must determine the kind of information needed (2,5,1)

(4) problems which require understanding of a mathematical term which is not represented by a pictorial model

Example 11. In which exercise below is 8 an addend?

a) $8 - 2 = 6$ c) $8 + 2 = 10$

b) $2 \times 4 = 8$ d) $8 \div 2 = 4$

Classification: Level 2 - understanding the term "addend" (2,4,12)

Example 12. Which of these objects is the best model of a cyclinder?

1) a soup can

2) an ice cream cone

3) a baseball

4) a phonograph record

Classification: Level 2 - understanding the term "cylinder". (2,13,12)

Example 13. In which number sentence is 6 a product?

a) $4 + 2 = 6$

b) $7 - 1 = 6$

c) $12 - 2 = 6$

d) $2 \times 3 = 6$

Classification: Level 2 - Understanding term "product" (2,4,12)

(5) Questions which students can answer by "understanding" some mathematical relationship or property rather than completing a calculation.

Note: For conventions applying to the factor II classification of such items, see p.31

Example 14. Which of the following make this number sentence true?

$3 \times (2 \times 4) = (3 \times 2) \times$ _____

a) 4

b) 6

c) 8

d) 21

Classification: Level 2 - a student who "understands" the associative principle can "observe that three times the product of 2, and 4 must equal the product of 3 and 2 times 4. (See Factor III, Level 13, p 40. (2,4,13)

Example 15. Which exercise below does not have the same product as the other three?

a)
$$\begin{array}{r} .23 \\ \times\ 12 \\ \hline \end{array}$$

b)
$$\begin{array}{r} 23 \\ \times\ 1.2 \\ \hline \end{array}$$

c)
$$\begin{array}{r} 2.3 \\ \times\ 1.2 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ .12 \\ \hline \end{array}$$

Classification: Level 2 - a student can "observe" the number of decimal places in the given numbers to identify "b" as the different product. (2,9,8)

Example 16. Which number makes (43 X 27) - 27 = _____ a true number sente

a) 1 c) 27

b) $\frac{43}{27}$ d) 43

Classification: Level 2 - a student who "understands" inverse operations can "observe" that multiplication and division by the same value results in no change in "43". (2,4,13)

Note: Special conventions for Factors III classification of items involving conceptual understanding (Levels 1 or 2) are found on page 36.

Levels 3 and 4 - Skills

Tasks which deal with the use of the algorithms of arithmetic, or fluency with mathematical symbols and illustrations will be classified at one of the skill levels. Again, two levels are distinguished by the presence (Level 3) or absence (Level 4) of pictures.

A. Level 3 - Skill in Reading Graphs, Tables, Etc.

This level describes the skill of reading mathematical diagrams, graphs tables, and measurement instruments. Problems of this type will be classified at Level 3 if the task involves.

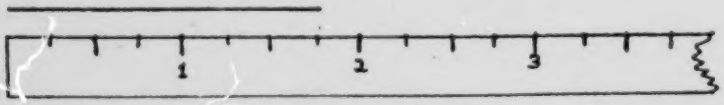
(1) only the identification of a specific value which is given in the illustration or,

(2) an appraisal of order or equivalence relationships which exist among values given in a table, graph, etc.

Convention:

If any calculation is required (e.g., adding, subtracting) the classification will be at Level 5 of Factor I.

Example 17. In the picture below, how long is the line segment above the ruler?



1) One and one-half inches

2) One and three-quarters inches

3) Two inches

4) Two and one-quarter inches

Classification - Level 3 - Reading a ruler (3,11,1)

Example 18. What time is shown by the clock below?

1) 3 minutes after 9

2) 15 minutes after 9

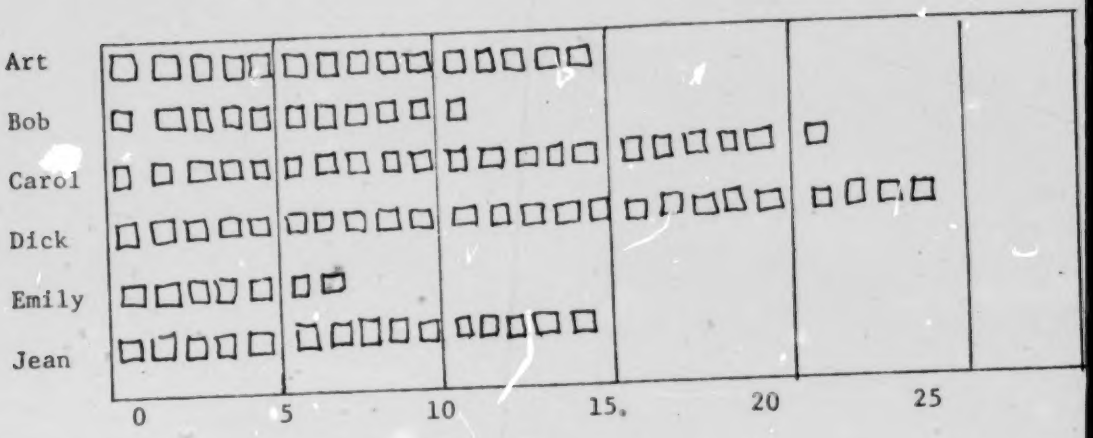
3) 9 minutes to 3

4) 14 minutes to 3

Classification: Level 3 - Clock reading (3,11,1)

Example 19 and 20.

Number of Stamps from Other Countries
Six Boys and Girls Have in Their Collection



Example 19. How many stamps from other countries does Bob have in his stamp collection?

a) 10 b) 11 c) 15 d) 25

Classification: Level 3 - reading a graph (3,3,1)

Example 20. Who has more stamps than Carol?

1) Art 3) Jean

2) Dick 4) Bob

Classification: Level 3 - requires finding a value which is greater than Carol's. (3,3,2)

B. Level 4 - Computational/Numberation Skills

The intent of much of the material in elementary school mathematics is to develop fluency in performing arithemtic operations or dealing with number systems. If the task is directed at proficient use of the skills of computation or numeration it will be classified at Level 4 of Factor I.

Numeration Skills, including

(1) the reading or writing of numbers in symbols or words,

(2) counting by 1's, 2's, 5's or 10's,

(3) naming equivalent fractions,

(4) or recognizing the order of numbers or giving a value greater than a give value will be classified at Level 4 of Factor I unless pictorial models are presented to develop understanding (Level 1)

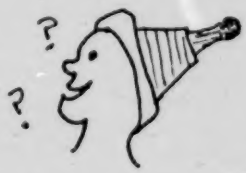
In some cases drawings are included to make the instructional material more attractive. If the illustrations are clearly "decorative" the pictures should be ignored and the unit will be classified at Level 4. Also, fraction and percent problems are frequently written without a multiplication symbol (e.g. ½ of 40). These will also be classified at Level 4.

Example 21.

$$\begin{array}{r} 33 \\ \times\ 24 \\ \hline \end{array}$$

a) 543 b) 682 c) 792 d) 972

Classification: Level 4 - calculation required (4,3,8)



Example 22. Two thousand eighty is written _____.

a) 2080 b) 2800 c) 2008 d) 2808

Classification: Level 4 - skill with number writing. (4,3,1)

Example 23. Which fraction is greater than 1/2?

a) 2/3 c) 3/6

b) 2/5 d) 1/4

Classification: Level 4 - skill with numeration. (4,7,2)

Example 24. What number comes next?

2, 4, 6, _____ 19

Classification: Level 4 - counting by two's. (4,1,2)

Convention:

Problems dealing with sequences of numbers that are NOT part of a regular sequence of counting by 1's, 2's, 5's, or 10's. (e.g. 2, 7, 12 or 6, 12, 18) will be classified at Level 6 of Factor I.

Levels 5 and 6 - Applications

Mathematics problems will be classified at one of the application levels if:

(1) the operation which must be performed is not explicitly stated but is implied by a statement of conditions, i.e., story problems and,

(2) the student must find the solution, not just a solution strategy as in Level 2. (See Example 9.)

Problems of this type usually involve the application of some mathematical concept and/or skill.

Convention:

If a problem is stated as an alegebraic sentence (e.g. 3 + N = 9) which must be solved for the unknown, the operation (although opposite the one given) is assumed to have been specified and the item is classified at Level 4. Such problems will not be classified at Level 6.

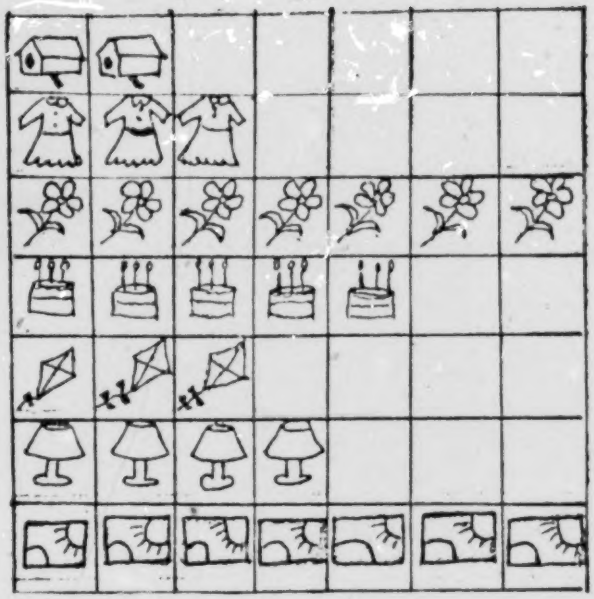
A. Level 5 - Applications involving graphs, tables, etc.

If the information for an application problem is presented in a graph table, diagram, picture, or the problem requires the use of a measurement instrument it will be classified at Level 5 of Factor I. The critical difference between skill level 3 and application Level 5 is that some calculation must be involved if the classification is at Level 5.

Example 25. The perimeter of the playground is _____ ft.

Classification: Level 5 - read diagram and compute "perimeter". (5,13,12)

Example 26. Hobby Contest Entries



How many more cakes were entered in the contest than birdhouses?

Classification: Level 5 - Read a graph and subtract. (5,1,6)

Example 27. Number of children using recreation areas.

	Mon	Tues	Wed	Thurs	Fri	Sat
Hobby and Paint Room	8	9	10	14	16	18
Outdoor Swimming Pool	15	18	16	0	16	20
Reading Room	3	3	4	11	4	2
Slides and Swings	10	15	11	0	9	17

Which part of the Center was used by the fewest children in the week?

a) Hobby and Paint Room

b) Reading Room

c) Slides and Swings

d) Swimming Pool

Classification: Level 5 - Read table, compute totals, and compare (5,3,5)

Level 6 - Application Without Graphs, Tables, Etc.

If the conditions of an application problem are met but the information is not presented in the form of a graph, table, or diagram and the use of measurement devices is not required, the unit will be classified at Level 6 of Factor I. "Story problems" are classified at the level if the student must find the final solution.

Example 28. Mrs. Sando bought 6 yards of fringe to trim the bottom of the curtains. She used only $4\frac{1}{2}$ yards. How many yards of fringe did she have left?

Classification: Level 6 - determine operation and compute (6,11,7)

Example 29. Find the missing number.

4 10 16 22 ?

Classification: Level 6 - determine operation and compute (6,3,4)

Note: Special conventions to describe the complete classification of problems dealing with sequences are found on page 18 (Factor II) and page ? (Factor III)

Factor II -- Nature of the Material

While Factor I was concerned with the general intent of the unit and how the material was presented to the student, Factor II is concerned with the nature of the material itself, i.e., does the problem deal with whole numbers, fractions, decimals, geometry, or measurement? Thirteen levels are defined in Factor II and a fourteenth level, "other", is included to accomodate the classification of material which may not be appropriately described by the specified levels.

Material in tasks which were classified at levels 1, 3, or 5 of Factor I (using pictures) will sometimes have implicit numerical value (e.g., a drawing is used to illustrate a fraction, or a numberline is used to model a sequence of numbers, or a picture graph conveys the numbers to be used). In such situations the nature of the "implied" material will be classified at Factor II*

In Factor II, the taxonomy forces what otherwise would be overlapping levels to be mutually exclusive by ranking the nominal characteristics in order of importance. "Importance" was based on two criteria. First, priority was given to the sequence of presentation observed in most elementary curricula, i.e., whole numbers are studied before fractions and fractional parts are taught before decimals are introduced.

Second, the levels were ordered to afford a more precise description of content when one level is embedded in another. Although most of the problems in elementary school mathematics will include quantities (e.g., whole numbers, fractions) in many instances these numbers are embedded in problems and serve only as a vehicle to quantify, rather than constituting the actual content.

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* Hence on page 5: Example 1 would be at Level 1 of Factor II.
Example 2 would be at Level 6 of Factor II.

For example, a problem might concern determining how many quarts there are in one gallon. The quantities here are all single digit whole numbers, but the problem actually deals with units of measurement.

While the rank ordering is admittedly subject to argument, it is an attempt to represent a unit to be classified by its single most demanding characteristic of Factor II. From most to least demanding material (excluding "other"), the levels of Factor II are: Geometry, Essential Units of Measurement, Measurement, Percents, Decimals, Mixed Numbers, Unlike Fractions, Single or Like Fractions, Algebraic Sentences/Phrases, Number Sentences/Phrases, Multiple Digit Whole Numbers, Single and Multiple Digit Whole Numbers, and Single Digit Numbers or Basic Number Facts.

In the following section the levels with the exception of "other", are described one at a time in descending order. This is done to facilitate parsimony of description; a unit is classified according to the most demanding material it contains.

Convention:

Problems involving sequences of numbers (e.g. 4, 10, 16, __, 28) will be classified according to all the numbers given. More than one multiple digit number is given, hence the classification for the example would be at Level 3 of Factor II, even though the solution can be found using combinations of single and multiple digit numbers. (e.g. 16 + 6).

Level 13 - Geometry

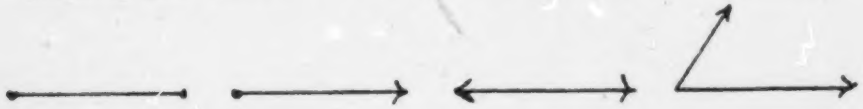
If an essential part of a problem is to identify a figure, term or property of geometry the appropriate classification is Level 13 of Factor II. This should not, however, be confused with the levels of Factor I which specify that a problem is presented with an illustration or model. For example, a problem can have some data presented in the form of a geometric figure, but the mathematical material needed for the solution of the problem is numerical and does not deal with the actual figure or its properties.

Example 30. Write the number.

Classification: Not Level 13 - Even though triangles are used to depict the number "7" the problem does not require the identification of the triangle. Hence, "Geometry" does not describe the nature of material. It is classified according to the single digit number illustrated, Level 1. (1, 1, 1)

Another distinction which should be made is the difference between "numberline" and "line segment." In general, the number line will be considered as a model of a number-related concept and should not be classified as "geometry." When "line segment" is used the material will usually apply to the geometric notion and be classified at Level 13 of Factor II. (For exception see p.22, Example 36.)

Example 31. Which of these represents a line segment?



Classification: Level 13 - Geometry (1, 13, 12)

Example 32.

The numberline above shows that 3 + 4 = _____.

a) 0 c) 7

b) 3 d) 10

Classification: Not Level 13 - Numberline is an illustration of numbers. (1,1,3)

One final distinction must be made for the classification of problems involving perimeter, area, and volume. Because the fundamental concepts of both geometry and measurement appear in elementary school materials, special consideration must be given to identify these categories. When the problems entail (a) finding the perimeter or area of a plane figure or (b) finding the surface area or volume of a solid, requiring the use of the properties of those figures i.e. using a formula, the classification will be at Level 13, Geometry.

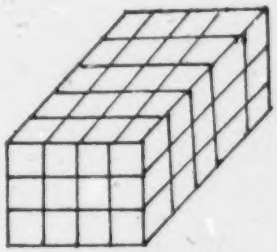
Example 33. The figure shown below is a rectangular prism. What is its volume?

a) 12 units

b) 20 units

c) 38 units

d) 60 units



Classification: Level 13 - Geometry* - the student must have an understanding of the geometric figure in order to find the volume of the figure pictured. (1,13,12)

Convention:

If the properties of the geometric figures are not essential to the solution and the task is meant to develop conceptual understanding of volume, or area, then the classification will not be Geometry (see page 6, Example 5 where the area can be found simply by counting the blocks.

Levels 12 and 11 - Measurement

If the units under analysis have not been classified as "Geometry" consideration must be given to the two measurement categories. These levels apply to the materials involving the measurement of various physical properties (temperature, distance, weight, and volume) as well as to the measurement of time and to some problems dealing with money. To further describe units classified in these measurement categories, the physical property that is measured should be recorded.

A. Level 12 - Essential Units of Measurement

This level is intended to identify materials that require knowledge of various essential units as part of a measurement system. Tasks which have not been classified at level 13 should be considered for level 12.

(1) Problems which deal with quantities expressed in different units of measurement in which the quantities must be transformed from one unit of measurement to another in order to solve the problem, are classified under Level 12 of Factor II, regardless of the type of quantities involved (e.g., whole numbers, fractions, or decimals).

Example 34. If candy costs 10¢ an ounce, what will a pound cost at that price?

a) $1.00 c) $1.25

b) $1.20 d) $1.60

Classification: Level 12 (weight) - one pound must be transformed to 16 oz. (6,12,8)

Example 35.

3 ft. 3 in.

\- 2 ft. 8 in.

Classification: Level 12 (length) must transform feet to inches before subtracting. (4,12,6)

(2) if a problem deals with the recognition of the physical or spatial equivalent of a measurement unit, then the item is classified under Level 12 of Factor II.

Example 36. Which line segment is nearest three inches long?

a) _____

b) _____

c) _____

d) _____

Classification: Level 12 (length) - Spatial equivalent of 3 inches (1,12,15)

Example 37. What metric unit should be used to measure the distance from Detroit to Cleveland?

a) mm b) km c) kg d) g

Classification: Level 12 (length) - recognition of spatial equivalent of units (2,12,12)

(3) if a problem deals with recognition of the money value of coins it will be classified at Level 12 of Factor II. (For other conventions referring to the classification of money problems see page 24 and 25)

Example 38. Art bought a 20 cent ice cream cone and paid for it with a quarter. How much change did he receive?

Classification: Level 12 (money) - a quarter is 25 cents (6,12,6)

(4) if a problem deals with time changes from a.m. to p.m. it will be classified at Level 12 of Factor II

Example 39. The Jones family left home at 11:00 a.m. They arrived at the camp site three hours later. What time did they arrive at the camp?

Classification: Level 12 (Time) - Change 14:00 to 2:00 p.m. (6,12,3)

The key to correctly classifying material as essential units of measurement is to determine if the use of the relationships between metrics (transformations) or an understanding of the actual dimension of a standard metric is essential to solving the problem.

B. Level 11 - Measurement

Items which have not been classified at Levels 13 or 12 should be considered for this level. Some materials in mathematics relate to measurement but do not require understanding the units of a standard system. The nature of material for such items will be described by Level 11, of Factor II if problems involve:

(1) reading clocks, ruler, or other measurement instruments (see Ex. 17 & 18)

Example 40. The time shown on the clock is ____.

a) 3:00 b) 12:00 c) 12:03 d) 3:15

Classification: Level 11 (time) (3,11,1)

(2) calculations with measurement quantities which do not involve transformation of essential units

Example 41. 5 ft. 4 in.
+ 1 ft. 5 in.

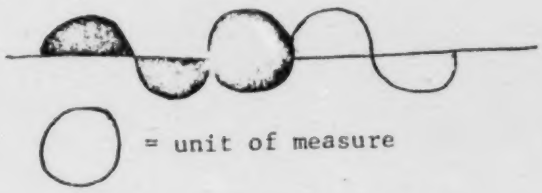
a) 6 ft. 9 in. c) 5 ft. 1 in.

b) 4 ft. 9 in. d) 6 ft. 1 in.

Classification: Level 11 (length) - no transformation of units required (4,11,3)

(3) the use of a nonstandard unit for measurement

Example 42. What is the area of the shaded part of the figure below?



Classification: Level 11 (area) - use of nonstandard unit (1,11,12)

(4) measurement of the length of a line segment or some actual measurement

Example 43. Use your ruler to find the width of your desk.

Classification: Level 11 (length) (3,11,1)

Convention

Problems dealing with coins or money will never be classified at Level 11. If the task does not require knowing the value of a coin (Level 12) it will be classified as a decimal or a whole number problem (see pages 25 and 33).

Level 10 - Percents

Items which have not been classified at Levels 13 through 11 will be classified at this level if one or more of the given quantities are given as percentages or if the answer is a percent.

Example 44. What percent of this region is shaded?

a) 50%

b) 10%

c) 25%

d) 100%

Classification: Level 10 (1,10,1)

Example 45. Which percent has the smallest value?

a) 85% c) 60%

b) 75% d) 55%

Classification: Level 10 - ordering percents (4,10,2)

Level 9 - Decimals

Items which have not been classified at Levels 13 through 10 will be classified at this level if one or more quantities are expressed in decimal form.

Convention

For problems involving money, assume students can convert cents to dollars (e.g., 59¢ = \$.59) and dollars to cents (e.g., \$5 = \$5.00). If such transformations are needed to solve the problem, it will be classified as decimals (Level 9) not as essential units of measurement as the coin problems were (see page 22). However, if all the information is given in cents (or dollars), and decimal notation is not required for the problem, classify the unit under one of the whole number levels (see descriptions of Levels 1 through 3).

Example 46.

$$\begin{array}{r} 48.3 \\ -\ 4.8 \\ \hline \end{array}$$

a) 4.35 c) 43.5

b) 4.46 d) 44.6

Classification: Level 9 - subtraction of decimals (4,9,7)

Example 47. Sue earned $4.50, $3.00, and $2.25 helping neighbors with yard work. How much did she earn altogether?

a) $10.75 c) $9.75

b) $9.00 d) $11.00

Classification: Level 9 (6,9,5)

Example 48. Joan bought a belt for $6.25. She gave the clerk a ten dollar bill. How much change should she get back?

a) $3.75 b) $4.75 c) $3.85 d) $4.85 (6,9,7)

Classification: Level 9 - This example involves decimal numeration.

Example 49. Harry bought a candy bar for 5¢ and bubble gum for 2¢. How much did he spend?

a) 7¢ b) 3¢ c) 10¢

Classification: Level 1 - decimal numeration is not required (see p.33) (6,1,3)

Levels 8, 7, and 6 - Fractions

Items which have not been classified at Levels 13 through 9 will be classified as fractions if (1) one or more of the given quantities is stated as a common fraction or (2) solution of the problem requires the formation of a fraction (mixed numbers are considered as fractions).

Level 8 - Mixed Numbers

A task will be classified at level 8 if at least one of the given quantitie is stated as a fraction greater than one. These will include mixed numbers (e.g., $3\frac{1}{2}$) and improper fractions (e.g., 4/3).

Convention:

The final answer will not be considered in determining the classification of fraction problems when the problem deals with addition, subtraction, multiplication or division of fractions.

Example 50. 8 1/2 - 5 3/4 = ?

a) 2 3/4 b) 3 3/4 c) 3 1/4 d) 2 1/4

Classification: Level 8 - two fractions are mixed numbers (4,8,7)

Example 51. 2 2/3 x 1/3 = ?

a) 8/9 b) 5/9 c) 2 2/3 d) 2 1/9

Classification: Level 8 - at least one mixed number is given (4,8,8)

Example 52. 8/3 = ?

a) 8 1/3 b) 2 1/3 c) 2 2/3 d) 1 1/3

Classification: Level 8 - given value is an improper fraction (4,8,1)

Level 7 - Unlike Fractions

If a problem involves fractions and has not been classified at Level 8 it should be considered for level 7. Problems will be classified at this level if they deal with more than one fraction and the denominators are not the same.

Convention:

Tasks which involve only finding an equivalent fraction with a different denominator will be classified at Level 7.

Example 53. 3/4 + 2/3 = ?

a) 5/7 b) 1 5/12 c) 1 1/4 d) 1 2/3

Classification: Level 7 - fractions do not have like denominators
The correct answer is a mixed number but the results of a computation are not considered when classifying fraction problems. (4,7,3)

Example 54. $1/3 \div 3/4 = ?$

a) 1/40 b) 4/9 c) 1 1/4 d) 1 1/3

Classification: Level 7 4,7,9)

Example 55. 2/3 _____ 1/2.

a) > b) < c) =

Classification: Level 7 (4,7,2)

Example 56. 2/3 = ____/12

a) 6 b) 8 c) 2 d) 3

Classification: Level 7 - finding equivalent fractions with different denominator (4,7,1)

Level 6 - Single or Like Fractions

If a problem has not been classified at levels 13 through 7 it should be considered for this level. Tasks will be classified at this level if:

(1) only one value is given in fraction form or,

(2) more than one fraction is given but each has the same denominator or,

(3) the task requires the formation of a fraction from given information.

Example 57. $3/8 + 1/8 = ?$

a) 1/2 b) 1/4 c) 3/16 d) 4/16

Classification: Level 6 - the answer is not considered for addition problems. (4,6,3)

Example 58. $3 \times 1/2 = ?$

a) 3/6 b) 1 1/2 c) 1/2 d) 9

Classification: Level 6 - only one fraction is given (4,6,8)

Example 59. What part of this figure is shaded?

a) 1/2 b) 1/3 c) 1/4 d) 1/5

Classification: Level 6 - the problem requires the formation of a fraction (1,6,1)

Example 60. There are 4 boys and 3 girls in the fourth grade class. What is the ratio of girls to boys?

a) 3/4 b) 1/4 c) 4/3

Classification: Level 6 - the problem requires the formation of a fraction, to express a ratio. (6,6,12)

Levels 5 and 4 - Sentences/Phrases

Material which has not been classified at Levels 13 through 6 should be considered for one of the "sentences/phrases" levels. Sometimes the tasks to be analyzed are more appropriately described by the "expressions" involved rather than by the quantities which are stated.

Number sentences most often deal with equality (=), but sometimes greater than (>) or less than (<) expressions are used. Such items will be classified at Level 5 or Level 4.

Convention

Greater than or less than expressions will be classified here only when a calculation is required on at least one side of the inequality. (see example 63). At Factor III the task will be classified according to the calculation performed.

A. Level 5 - Algebraic Sentences/Phrases

This level describes sentences or phrases which are algebraic in nature, i.e., they deal with missing values or variables. Tasks will be classified at Level 5 if:

(1) The given numerical expression is a sentence involving one or more unknowns with at least one unknown not isolated on one side of the equal sign. Missing addend and missing factor problems are the most frequently occuring examples.

Example 61. $7 - ____ = 2$

a) 9 b) 5 c) 6 d) 14

Classification: Level 5 - expression given with missing value (4,5,6)

Example 62. $6 \times ____ = 18$

a) 12 b) 2 c) 3 d) 24

Classification: Level 5 - missing factor (4,5,9)

Example 63. Find the largest number that will complete the inequality:

$$3 \times \square < 16$$

Classification: Level 5 - Inequality with missing factor. (4,5,10)

Convention:

When a task requires the selection of an algebraic or number sentence which meets or suits certain conditions the correct answer will be used to determine the classification of the task at Factor II and Factor III.

Example 64. On Saturday two parking lots were full. There were 36 cars in all. There were 19 cars in one lot. How many were in the other one?

a) _____ x 19 = 36

b) 19 + _____ = 36

c) 36 + 19 =

d) 36 ÷ _____ = 19

Classification: Level 5 - correct answer (b) is an algebraic sentence (2,5,7)

Convention:

Problems which deal with conceptual understanding of properties (see page13) will not be classified as algebraic sentences (Level 15) unless letters are used in the statements. They will be classified as number sentences (Level 4) when boxes or blanks are used in the statement. (See Example 14)

Example 65. If 19 - a = b, then _____

a) b + a = 19

c) b x a = 19

b) a ÷ b = 19

d) b x 19 = a

Classification: Level 5 = Conceptual understanding of properties with letters in the expressions. (2,5,13)

(2) the task requires the identification of a missing variable which is needed to solve a problem.

Example 66. What will 30 books cost for one class? You can answer this if you know:

a) the size of the class

c) which room will use them

b) the names of the books

d) the cost of each book

e) who sells the book

Classification: Level 5 - must state the missing variable (2,5,1)

B. Level 4 - Number Sentences/Phrases

Materials which have not been classified at Level 5 but are best described by the "expressions" involved will be classified at Level 4. Tasks will be classified at this level if:

(1) the answer to a problem is expressed as a number sentence or phrase (that is not algebraic) and not as a solution to that number sentece

Example 67. 908 = ____?

a) (9 x 100) + (8 x 10)

b) (9 x 100) + (8 x 1)

Classification: Level 4 - the answer is a number phrase (2,4,14)

Example 68. 7 + (5 + 4) = ____

a) (7 + 5) + (7 + 4)

b) (7 + 5) + 9

c) (3 + 4) + (3 + 2)

d) (7 + 5) + 4

Classification: Level 4 - the answer is a number phrase (2,4,13)

Example 69. There were 13 children waiting to cross the road. Eight of them ran across. How many were still waiting to cross?

a) ____ x 8 = 13

b) 13 - 8 = ____

c) 13 + ____ = 8

d) 13 ÷ 8 = ____

Classification: Level 4 - the correct answer is not an algebraic sentence (2,4,6)

(2) The problem requires only an identification of the operations required to solve a problem. It will be classified at Level 4 of Factor II for the correct answer deals with information needed to form a number sentence.

Example 70. What should replace the ____ in the number sentence 15 ____ 3 = 5?

a) + b) - c) x d) ÷

Classification: Level 4 - must specify the operation needed (2,4,9)

(3) More than one different mathematical operation is required and the order of calculation is (a) essential to the solution or (b) explicitly stated by the presence of parentheses, the material should be classified at Level 4.

Example 71. $4 + (3 \times 4) =$ ____

a) 16 b) 15 c) 11 d) 28

Classification: Level 4 - problem stated using parentheses (4,4,11)

(4) A solution is required but "observation" of the expression will supply the answer without actual completion of all the specified calculations (see convention on page 30).

Levels 3, 2, 1 - Whole Numbers

Items which have not been classified at levels 13 through 4 should be considered for one of the whole number categories. Money problems which have not been classified at Level 12 (see page 22) or Level 9 (see page 25) will be classified according to the numerical values stated, ignoring the money connotation.

When problems deal with more than one calculation, i.e., column addition, the classification is determined by the numbers given. The answers to intermediate calculations will not be considered, for Factor II Classification.

A. Level 3 - Multiple-Digit Whole Numbers

If a task has not been classified in levels 13 through 4 it will be classified at Level 3 if:

(1) it involves the recognition of place value of a whole number

Example 72. The 3 in 437 has a value of

a) 300 b) 30 c) 3

Classification: Level 3 - place value in a multiple digit number(2,3,14)

(2) the problems deal with more than one quantity stated as a multiple digit number or the only number stated is a multiple digit number

Example 73. 25 + 36 = ____

Classification: Level 3 - more than one multiple digit number (4,3,4)

Example 74. 15¢ + 12¢ = ____

a) 27¢ b) 3¢ c) 37¢

Classification: Level 3 - money units but decimal notation is not used (4,3,3)

Example 75. 45 + 8 + 12 = ____?

Classification: Level 3 - more than one multiple digit number, the single digit value is not considered (4,3,5)

B. Level 2 - Single and Multiple Digit Numbers

If a task has not been classified at Levels 13 through 3 it will be classified at Level 2 when two conditions are met.

(1) The given quantities include a single digit number and one and only one multiple digit number (if more than one given value is a multiple digit number the classification will be Level 3).

(2) For subtraction and division problems the calculation must not be a basic number fact, i.e., 17 - 9 = ? or 72 ÷ 8 = ?

Example 76. 45 x 3 = ?

a) 125 b) 135 c) 115 d) 171

Classification: Level 2 (4,2,8)

Example 77. 36 ÷ 2 = ____

a) 12 b) 13 c) 15 d) 18

Classification: Level 2 - Single-multiple digits and not basic number facts (4,2,9)

Example 78. Susan had 14 pine cones. She gave Carole 2 and Patty 4. How many does Susan have now?

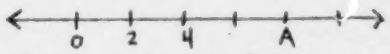
Classification: Level 2 - "14" is the only multiple digit number stated (6,2,11)

C. Level 1 - Single Digit Numbers or Basic Facts

If material has not been classified at Levels 13 through 2 it should be considered for Level 1. Two types of tasks will be classified at this level.

(1) when computations are not required and the given numbers are less than ten

Example 79.



On the number line above, A is equal to:

a) 6 b) 7 c) 8

Classification: Level 1 - single digit numbers (1,1,1)

(2) When computations entail only single digit numbers or the knowledge of basic number facts. The basic number facts are:

a) Addition and multiplication: stated numbers are less than 10

(9 + 9 = ____) (3 x 8 = ____)

b) Subtraction: one number is less than 10, the other is less than 19, and the answer is less than 10.

(18 - 9 = ____)

c) Division: the answer and the divisor are whole numbers less than 10. There is no remainder

(36 ÷ 9 = ____)

Level 14 - Other

Items which have not been classified at Levels 13 through 1 will be classified at Level 14. Units dealing with number systems in different bases, Roman numerals and, sets containing objects or letters and not referring to the number of elements are sometimes found. These units will be classifed at Level 14 and a description of the nature of material should be given.

Factor III - Operations

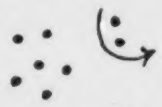
Factor I dealt with the general intent of the task and how the material was presented, while Factor II defined the nature of the material itself. Factor III concerns the type of operation which must be applied to the material. There are 15 levels of this factor. Levels 3 through 11 deal with ordinary arithmetic operations (e.g., add, subtract). The remaining levels deal with other "operations" a student might be required to perform in studying mathematics (e.g., order, estimate). As was done in Factor II, the levels have been ranked in order of importance for classification purposes. Again, the usual order in which operations are taught was considered when ranking the levels. Also, the levels describing concepts, properties, place value, and estimation were given priority, thereby eliminating the ambiguity when arithmetic operations are a part of units dealing with these content areas. In the following section the levels are described in descending order to facilitate parsimony of description. A unit is classified according to its most demanding characteristic.

One further clarification is needed for levels dealing with arithmetic operations (levels 3 through 11). Generally, tasks classified at these levels will require the completion of a calculation. The only exception to this will be those units which are classified in Levels 1 and 2 of Factor I.

Convention:

If problems deal with conceptual understanding of an arithmetic operation (e.g., multiplication) they will be classified according to the operation even though the student does not have to complete the calculation.

Example 80. This was one set. Now it is two sets. This is what happened.



a) $2 + 6 = 8$ c) $6 + 2 = 8$

b) $8 - 2 = 6$ d) $6 - 2 = 4$

Classification: Level 6 - Subtraction (see p. 46) (1,4,6)

Convention:

Problems which deal with sequences of numbers that are not the counting sequences by 1's, 2's, 5's or 10's will be classified at Factor III according to the operation which would be performed to generate the sequence "left to right." Hence: 4, 9 , 14, 19, 24, - would be classified as addition because you must add to generate sequential numbers. It would be classified as addition with carrying

Level 15 - Estimate

A unit is classified at this level if it requires an estimation of a value; in other words, the expected answer is an approximation rather than a precise statement of a quantity. There will be three types of estimates included.

(1) those dealing with approximations of numbers

Example 81. Round 4.84 to the nearest tenth.

a) 4.9 b) 4.8 c) 5.0 d) 5.8 (4,9,15)

Classification: Level 15 - approximate value of a number.

(2) those dealing with estimations of a measurement (see convention p. 38)

Example 82. A package that is heavy to carry might weight _____.

a) 500 g. b) 5000 mg. c) 50 Kg.

Classification: Level 15 - estimate the value of a measurement (2,12,15)

(3) those dealing with approximate calculations

Example 83. Which of the following is closest in value to 398 x 90?

a) 30,000 b) 40,000 c) 3000 d) 4000

Classification: Level 15 - approximate answer to a calculation (4,3,15)

Convention:

In the strict sense all measurements of physical properties are estimations of a true value. To eliminate ambiguity only those tasks which require the statement of a measurement value without the use of a measurement instrument or given measurement quantity will be classified as estimations (Level 15).

Level 14 - Interpret Place Value

If a unit has not been classified at Level 15 it should be considered for this level. A task is classified at Level 14 if the primary purpose of the item is to determine whether the student can recognize the different values digits have when placed in different positions within a numeral. Problems of this type will include those dealing with:

(1) expanded notation (see Example 67)

(2) the recognition of the place value of a particular digit or digits in a numeral

(3) addition problems, when addends each contain a single nonzero digit and those nonzero digits vary dramatically as to their assumed numerical value

Convention

The reading of numbers in verbal (e.g., thirty-two) or symbolic (e.g., 32) form will not be classified as place value. Such tasks will be classified as Identify equivalents (see page 51, rule 2).

Example 84. In which numeral is 5 in both the hundred place and the ones place?

a) 4,755 c) 4,557

b) 4,575 d) 5,475

Classification: Level 14 - Recognize place value of "5" (2,3,14)

Convention:

The Factor I classification of place value problems will always be at one of the levels of conceptual understanding (Levels 1 or 2)

Example 85. Which number sentence below is false?

a) 3 tens + 8 ones = 8 tens and 3 ones

b) 30 + 9 = 20 + 19

c) 2 tens + 15 ones = 3 tens + 5 ones

d) 5 tens - 5 ones = 4 tens + 5 ones

Classification: Level 14 - recognize place value (2,4,14)

Example 86. 80,000 + 5,000 + 40 = ____

a) 85,040 c) 80,540

b) 85,400 d) 854

Classification: Level 14 - the key to solution involves recognition of the place value of nonzero digit (2,3,14)

Level 13 - Recognize Properties

If an item has not been classified at levels 15 or 14, it should be considered for this level. Some problems in elementary school mathematics emphasize the recognition of the properties of arithmetic operations rather than the completion of the actual calculations. These properties are (1) the commutative property of addition or multiplication (e.g., $3 + 5 = 5 + 3$); (2) the associative property of multiplication or addition (e.g., $(2 + 5) + 7 = 2 + (5 + 7)$; (3) the distributive property of multiplication over addition (e.g., $3(5 + 2) + (3 \times 5) + (3 \times 2)$; (4) the property of inverse operations. This final property will be restricted to apply only to units which require the recognition of the inverse of an operation. It will not apply to the use of inverses when dividing fractional numbers or the use of inverse operations to solve equations. (e.g., _____ + 8 = 11 -- Classify as subtraction).

Example 87. Which of the following makes this number sentence true?

$8 + 6 = 6 + \square$ a) 6 b) 8 c) 12 d) 14

Classification: Level 13 - commutative property (2,4,13)

Example 88. What should replace the blank in the number sentence $8 \times 7 = (____ + 3) \times 8$?

a) 3 b) 4 c) 7 d) 8

Classification: Level 13 - distributive and commutative (2,4,13)

Example 89. $75 \times 8 - 8 =$

Classification: Level 13 - inverse operations (If the student recognized that multiplication and division are inverse operations the answer is apparent without calculations.) (2,4,13)

Level 12 - Apply Concepts (Terms)

If an item has not been classified at Levels 15 through 13, it should be considered for this level. Two kinds of units are classified at Level 12:

(1) problems which ask for a definition or example of the term only

(2) problems where a term, and the term alone (see example 89) suggests the arithmetic operation to be performed, i.e., "perimeter", "area", "average", "sum", "ratio".

Convention:

Any term which names, a level of some factor in the taxonomy will not be classified as a "term". Because Levels 1 and 2 are designated for "ordering" and "equivalents" these words will not be considered as terms under rule 2. Words referring to place value (e.g., tens place) or specific fractions (e.g. fourths) will not be classified as terms.

Convention

All term problems which ask for a definition or example of a term will be classified at one of the levels of conceptual understanding in Factor I. If a term suggests a calculation which must be performed and the student must perform the calculation the task will be classified at one of the application levels at Factor I.

The following list is designed to aid in deciding if the problem involves the interpretation of a term or is merely a situation where the term is included but its interpretation is not the primary task in the problem.

Terms

(1) Geometry: When the nature of the task is to identify an example of the specified term. Sphere, Circle, Diagonal, Diameter, Line Segment, Altitude, Intersection of Lines.

(2) Terms which imply a computational procedure in the given context. Perimeter, Area, Ratio, Circumference, Greatest Common Factor, Least Common Multiple, Prime Factors.

(3) Terms referring to operations on sets or set concepts. Empty Set, Union, Intersection.

(4) Terms given to identify elements in a computational problem (when the term is essential to the interpretation of the task). Addend, Divisor, Sum, Product, Quotient.

(5) Terms related to number systems (excluding place value terms). Prime, Composite, Even Number, Odd Number, Negative Number.

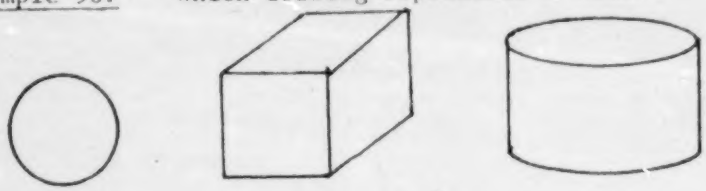
Convention :

Units of measurement in a standard system will not be considered as terms unless:

(1) the task requires the student to understand the unit as a "spatially" distinct metric (see example 37).

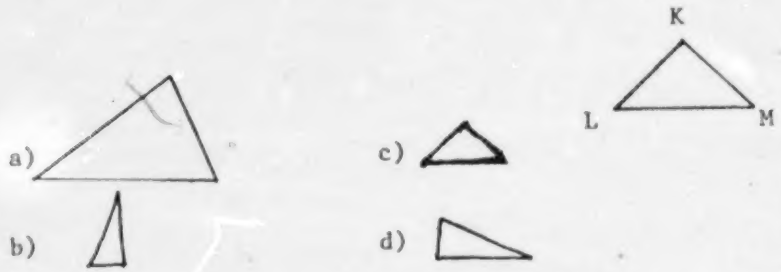
(2) the item deals only with explicit terminology and symbols in a system (e.g., Km = kilometer).

Example 90. Which drawing represents a cube?



Classification: Level 12 - select illustration of "cube". (1,13,12)

Example 91. Which triangle is similar to triangle KLM?



Classification: Level 12 must know the concept of "similar" (1,13,12)

Example 92. Three boys were 58, 61, and 66 inches tall. To find their average height, you can:

a) use 61 as the average

b) add the heights together

c) add the heights and multiply by 3

d) add the heights and divide by 3

Classification: Level 12 - focus on the definition of "average" as it applies to the situation (2,11,12)

Example 93. What is the sum of 2 and 6?

a) 12 b) 4 c) 3 d) 8

Classification: Level 12 - "sum" suggests the arithmetic operation (Note that if the expression 2 + 6 had been used, this would not have been classified at Level 12.) (2, 1, 12)

Level 11 - Combination

If a unit has not been classified at Levels 15 through 12, it should be considered for this level. Level 11 is for items in which more than one different arithmetic operation (Add, Subtract, Multiply, Divide) is performed. Ordering will not be considered as an arithmetic operation, i.e., if the task requires subtraction and the ordering of numbers it will not be classified as a combination operation. When classifying "story problems" that could be done by repeating an operation several times or by using two different operation, the most straightforward approach should be classified, unless there is some suggestion to a different approach.

Example 94. In servicing Mr. Lane's car the attendant used 5 quarts of oil at 65¢ a quart and 15 gallons of gasoline at 38¢ a gallon. What was the total cost for oil and gas?

a) $3.25 c) $8.95

b) $5.70 d) (not given)

Classification: Level 11 - even though the problem could be solved by adding only (add 65¢, 5 times and 38¢, 15 times), a combination of operations seems the likely strategy at Level 11 (6,11,11)

Convention

If a problem involves the conversion of units of measurement or the use of equivalent fractions, these procedures will NOT be considered in determining that the problem requires a combination of operations.

Example 95. Nancy bought 2 yards of ribbon. She used 5 feet to wrap some packages. How much ribbon does she have left?

a) 3 ft. b) 3 yd. c) 1 ft. d) 1 yd.

Classification: Not Level 11 - even though a multiplication was required to transform 2 yards into 6 ft., this was described at Factor II and only the subtraction operation is considered for Factor III classification. (6,12,6)

Example 96. $3\frac{1}{4} \div \frac{1}{2} =$ ____

Classification: Not level 11 - the transformation of $3\frac{1}{4}$ into 13/4 is not considered in determining the Factor III classification. (4,8,9)

Level 10 - Divide With Remainder

If a unit has not been classified at Levels 15 through 11, it should be considered for this level. A problem will be classified at Level 10 if it requires division which does not come out evenly or if it deals with the conceptual understanding of division with a remainder. Division of fractions will not be classified at Level 10.

Example 97. $9\overline{)183}$

a) 2R3 b) 9R3 c) 12R3 d) 20R3

Classification: Level 10 (4,2,10)

Example 98. $4\overline{)83}$

a) 20 b) 20 3/4 c) 22 d) 21

Classification: Level 10 - Even though the quotient is expressed using a fraction the division did not come out evenly (4,2,10)

Level 9 - Divide Without Remainder

If a unit has not been classified at Levels 15 through 10, it should be considered for this level. If a problem deals with (a) division of fractions or (b) division that does not involve a remainder or (c) the conceptual understanding of division, it will be classified at Level 9. Division which might be required to find an equivalent fraction (e.g., 2/4 = ___?___/2) will not be classified at this level.

Example 99. If stamps are 10¢ each, how many can you buy for 40¢?

a) 50 c) 10

b) 30 d) 4

Classification: Level 9 (6,3,9)

Example 100. $4 \div \frac{1}{2} =$ ____

a) 1/8 b) 1 c) 2 d) 8

Classification: Level 9 - all division of fractions will be at this level (4,6,9)

Example 101. 4 x ____ = 32

Classification: Level 9 - even though this is stated as a multiplication problem, division would be used to find the answer (4,5,9)

Level 8 - Multiply

If a unit has not been classified at Levels 15 through 9, it should be considered for this level. The multiplication to find equivalent fractions (e.g., $\frac{1}{2}$ = ____/4) or equivalent units of measurement (e.g., 2 ft. = ___?___ in.) will not be classified at Level 8. Other problems which require multiplication or the conceptual understanding of multiplication will be classified at Level 8.

Example 102. 3,057
x 6

a) 18,013 c) 18,432

b) 18,342 d) 183,042

Classification: Level 8 (4,2,8)

Example 103. The missing number is ____.

1 3 9 ____

a) 12 b) 18 c) 27

Classification: Level 8 - (see page 37) (6,1,8)

Example 104. Which sentence does this array mean?

. . . . . .
. . . . . .
. . . . . .
. . . . . .

e) 3 x 8 = 24 g) 4 x 6 = 24

f) 2 x 12 = 24 h) 12 + 12 = 24

Classification: Level 8 - conceptual understanding of multiplication (1,4,8)

Level 7 - Subtract With Borrowing

If a unit has not been classified at Levels 15 through 8, it should be considered for this level. Subtraction problems which require borrowing or tasks which focus upon conceptual understanding of subtraction with borrowing, will be classified at Level 7. Such problems include borrowing in the subtraction of whole numbers, decimal or fractions. They do not, however, include the subtraction facts, i.e., 17 - 9 = ____ (see Factor II, Level 1).

Convention

Measurement problems which require borrowing (see page 21 ex.35) will not be classified as subtraction with borrowing since this process is described when the items are classified as essential units at Factor II.

Example 105.
$$\begin{array}{r} 1204 \\ -\ 978 \\ \hline \end{array}$$

a) 225 c) 336

b) 326 d) 226

Classification: Level 7 (4,3,7)

Example 106.
$$\begin{array}{r} 5\ 1/3 \\ -\ 2\ 1/2 \\ \hline \end{array}$$

a) 2 5/6 b) 3 5/6 c) 3 1/2 d) 4

Classification: Level 7 (4,8,7)

Level 6 - Subtraction Without Borrowing

If a unit has not been classified at Levels 15 through 7, it should be considered for this level. If a problem deals with conceptual understanding of subtraction or requires subtraction for solution, but does not entail borrowing, it will be classified at Level 6.

Example 107. The next Saturday, Clay parked 14 cars in his yard, and John parked 5 in his. How many more cards did Clay park than John that day?

a) 9 c) 11

b) 19 d) NG

Classification: Level 6. (6,1,6)

Example 108.
$$\begin{array}{r} 8\ 3/4 \\ -\ 3\ 1/2 \\ \hline \end{array}$$

53

a) 4 1/4 b) 5 1/4 c) 4 2/4 d) 5 1/2

Classification: Level 6. (4,8,6)

Level 5 - Column Addition

If materials have not been classified at Levels 15 through 6 they should be considered for Level 5. When the task requires the addition of more than two numbers it will be classified at this level.

Example 109. $3.45 + $.25 + $1.10 = ____

a) $6.40 b) $4.80 c) $6.15

Classification: Level 5 - addition of more than two numbers (4,9,5)

Level 4 - Add With Carrying

If a unit has not been classified at Levels 15 through 5, it should be considered for this level. If the addition does not involve a basic number fact, i.e., 7 + 5 = ____, and requires addition with carrying or conceptual understanding of that process, it will be classified at Level 4.

Example 110.
$$\begin{array}{r} 38 \\ +\ 47 \\ \hline \end{array}$$

a) 715 b) 85 c) 71 d) NG

Classification: Level 4. (4,3,4)

Level 3 - Add Without Carrying

If a unit has not been classified at Levels 15 through 4, it should be considered for this level. If a problem deals with basic addition facts or addition without carrying or conceptual understanding of the addition operation, it should be classified at Level 3.

Example 111.
$$\begin{array}{r} 14 \\ +\ 12 \\ \hline \end{array}$$

a) 26 b) 28 c) 6 d) 18

Classification: Level 3. (4,3,3)

Example 112. ____ - 7 = 8

Classification: Level 3 - the answer is found by adding

Example 113. The missing number is ____.

3 6 9 ____ 15

a) 10 b) 12 c) 13 d) 14

Classification: Level 3 - 3 must be added to each number to get the next one (6,2,3)

Level 2 - Order

If an item has not been classified in Levels 15 through 3, it will be classified at Level 2 if it involves the ordering of values or numbers. The following types of problems will be considered to involve ordering (1) putting numbers in the correct order, (2) recognition of "greater than" or "less than" relationships, (3) understanding or ordinal numbers (e.g., first, second), (4) recognition of counting sequences or other numerical or nonnumerical patterns, and (5) tasks dealing with counting by 1's, 2's, 5's, and 10's will be classified under Level 2. Others will be under addition (Example 113).

Example 114 Which duck is third in line from the left?

a) b) c) d)

Classification: Level 2 - ordinal numbers (1,1,2)

Example 115. Which fraction is greater than 2/3?

a) 1/2 b) 3/5 c) 3/4 d) 4/9

Classification: Level 2 - "greater than" relationship (4,7,2)

Example 116. Which drawing would be next?

a) b) c) d) not given

Classification: Level 2 - pattern recognition (1,14,2)

Level 1 - Identify Equivalents

If a unit has not been classified at Levels 15 through 2 it should be considered for this level.

Items are classified at Level 1 of Factor III if the solution to the problem demands recognition of an alternative but equivalent way to represent the material contained in the item. Another way of stating this is that the solution to the problem is an expression or quantity contained in the item but is expressed in a different way (or form). Several examples of equivalent forms are given in the following table.

Form of the material as given in the item	Equivalent form as given in the solutio
1. Graphical or pictorial (including pictorial representations of geometric figures or sets)	Numbers representing the graphical material
2. Numbers expressed in words (e.g., ninety-two)	The corresponding numbers themselves (e.g., 92)
3. Measurement given in one metric (e.g., 1 yd.)	Statement of equivalent metric (e.g., 36 in.)
4. Number sentences	Alternative number sentences equivalent in value not dealing with defined properties (see Level 13)
5. Numbers expressed in other number systems (e.g., Roman numerals such as VI)	The same number expressed in Arabic form (e.g., 6) (Note: While these would be classified as "other" at Factor II. They should be classified at Factors I & III)
6. Fraction value	equivalent fraction

Example 117. Three of these are equal. Which one is not equal to the other three?

a) $7 + ___ = 12$ b) $12 - 7 = ___$

c) $12 + 7 = ___$ d) $___ + 7 = ___$

Classification: Level 1 - equivalent number sentence

Example 118. What fraction of the figure is shaded?

a) 2/4 b) 2/6 c) 1/4/ d) 4/6

Classification: Level 1 - number represented in a picture

Summary

Factor I - General Intent and Mode of Presentation (p.4)

Level 1: Pictorial or concrete models illustrate a concept. (p.4)

Level 2: Goal is understanding but pictures not used. (p.7)
These usually do not involve finding the answer to a calculation. They require:

a) conclusions about an operation (p.7)

b) finding a number sentence that meets given conditions (p.7)

c) making an observation about a number sentence (p.8)

d) illustration of a term (p.8)

e) place value problems (p.5)

Level 3: Reading graphs, tables, diagrams, measurement instruments (p.10)
Convention - no calculations involved (p.11)

Level 4: Skills with numbers - (p.12)

Reading or writing or ordering (p.13)

Computational skills - operation is specified (p.13)

Convention: Counting by 1's, 2's, 5's, and 10's (p.14)

Level 5: Applications with graphs, tables, measurement instruments (p.14)

a) information given in pictorial form (p.14)

b) calculation required (p.14)

Level 6: Applications without graphs, tables, measurement instruments (p.16)

a) word problems (require a solution) (p.16)

b) number sequence other than counting sequences (p.14)

c) operation not specified (p.14)

Factor II - Nature of Material (p.17)

(Choose highest number which describes the unit)

Level 13: Geometry (p.18)

a) line segment - not a number line (p.19)

b) geometric figures and terms (p.41)

Convention: Volume, Area, Perimeter (p.20

Level 12: Essential Units of Measurement (p.21)

a) units of a standard system must be transformed as perceived (p.21)

b) distance, temperature, weight, mass (p.21)

c) money - "coin" problems (p.22)

d) time - a.m. to p.m. (p.22)

Level 11: Measurement

a) knowledge of units of a standard system not required (p.23)

b) reading measurement instruments (p.23)

Convention: No money problems (p.24)

Level 10: Percents (p.24)

Level 9: Decimals (p. 25)

Convention for money problems (p.25)

Level 8: Mixed Numbers (p.26)

a) mixed numbers (p.26)

b) improper fractions (p.26)

Convention: Answer to calculation not considered (p.27)

Level 7: Unlike Fractions (p.27)

a) more than one fraction and denominators are different (p.26)

b) finding equivalent fraction with different denominator (p.26)

Convention about answer (see above) (p.27)

Level 6: Single or Like Fractions (p.28)

a) only one fraction given (p.28)

b) all given fractions have the same denominator (p.28)

c) form a ratio (p.28)

d) name fractional part in given picture (p.28)

Convention about answers see above (p.27)

Level 5: Algebraic sentences/phrases (p.29)

a) unknown not isolated by equal sign (p.30)

b) missing value must be identified (p.31)

c) answer is an algebriac sentence (p.30)

Convention for inequalities (p.31)

Convention - no conceptual understanding problems (p.31)

Level 4: Number sentences/phrases (p.32)

a) answer is a number sentence (p.32)

b) operation must be identified (p.32)

c) more than one different calculation and order is important (p.33)

d) answer can be "observed" without calculation (p.33)

Level 3: Multiple digit whole numbers (p.33)

a) only one number given and it has multiple digits (some place values) (p.33)

b) more than one number has multiple digits (p.33)

Level 2: Single and Multiple Digits (p.34)

a) the problem is <u>not</u> a basic number fact (p.34)

b) two or more numbers given but only one has multiple digits (p.34)

Level 1: Single Digit or Basic Number Facts (p.35)

a) only single digit numbers given (p.35)

b) basic number facts (p.35)

Level 14: Other (p.35)

Factor III - <u>Operations</u> (p.36)

(Choose the highest number which describes the unit)

Level 15: Estimate (p.37)

a) rounding (p.37)

b) estimate a measurement value (p.38)

c) approximate calculations (p.38)

Level 14: Place Value (p.38)

a) expanded notation (p.38)

b) place value of a digit (p.38)

c) addition problems with only one nonzero element in addends (p.38)

Level 13: Recognize Properties (p.40)

a) commutative (p.40)

b) associative (p.40)

c) distributive (p.40)

d) inverse operations (p.40)

Level 12: Apply Concepts (terms) (p.40)

a) definitions (p.40)

b) term suggests arithmetic operation (p.40)

Conventions for order and place value (p.41)
for measurement units (p.42)

Level 11: Combination (p.43)

more than one different operation (p.43)

ordering fraction transformations and measurement conversions
not considered (p.43)

Level 10: Divide with remainder (p.44)

a) division does not come out evenly (p.44)

b) conceptual understanding (p.36)

c) no fraction problems (p.44)

Level 9: Division without remainder (p.45)

a) division of fractions (p.45)

b) division comes out evenly (p.45)

c) conceptual understanding (p.36)

d) missing factor problems (p.45)

Level 8: Multiply (p.45)

a) conceptual understanding (p.36)

b) not equivalent fractions or to transform units of measurement (p.4

Level 7: Subtraction with borrowing (p.46)

a) must borrow to subtract whole numbers (p.46)

b) " " " " mixed numbers (p.46)

Convention - not borrowing for measurement units (p.47)

Level 6: Subtraction without borrowing (p.47)

subtraction problems (p.47)

missing addend problems (p.47)

Level 5: Column addition (p.48)

more than two addends (p.48)

Level 4: Add with carrying (p.48)

Level 3: Add without carrying (p.48)

Level 2: Order (p.49)

a) put numbers in order (p.49)

b) recognize "greater than" or "less than" values (p.49)

c) ordinal numbers (p.49)

d) counting sequences (p.49)

e) other patterns (p.49)

Level 1: Identify equivalents (p.50-51)

Sample Exercises A

#1 If the bundles have ten sticks each what number is represented?

a) 32 b) 203 c) 23 d) 33

#2 Mary bought some candy that cost 12 cents. She gave the clerk a quarter. How much change should she get back?

#3

Pets brought to school	
Cats	4
Turtles	2
Rabbits	2
Dogs	3

The table shows the number of pets the third graders had in the pet show. How many pets were there all together?

#4 $36 \div 9 =$

#5 A national park is 450 miles from our home. If we go 240 miles the first day, how many miles from the park will we be ?

#6 What part of the figure is shaded?

a) 1/2 b) 1/3 c) 1/4 d 1/5

#7 $1/5 - 1/6 =$

#8 Which sentence does this array mean?

a) $2 \times 3 = 6$ b) $2 + 3 = 5$ c) $5 \times 1 = 6$ d) $1 \times 5 = 6$

#9 What time is shown on the clock?

#10 $3 + (4 + 8) = (3 + __) + 8$

a) 4 b) 3 c) 7 d) 8

11# Scores on Math Test

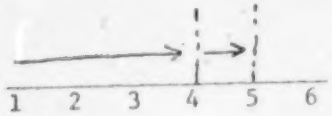
Sue	75%
Joe	45%
John	80%
Mary	72%
Lee	95%

Which student got the most problems correct on the math test?

12# In which numeral is 5 in both the tens place and the hundreds place?

a) 4755 b) 4557 c) 5457 d) 4575

#13



The number line shows that

4 + 1 = ___

#14 If candy costs 25¢ an ounce, how much will a pound cost?

#15 Which of the following is a triangle.

a) b) c)

Classification for Sample Exercises A

#1 (1,3,14)

#2 (6,12,6)

#3 (5,1,5)

#4 (4,1,9)

#5 (6,11,6)

#6 (1,6,1)

#7 (4,7,9)

#8 (1,4,8)

#9 (3,11,1)

#10 (2,4,13)

#11 (3,10,2)

#12 (2,3,14)

#13 (1,1,3)

#14 (6,12,8)

#15 (1,13,12)

Sample Exercises: B

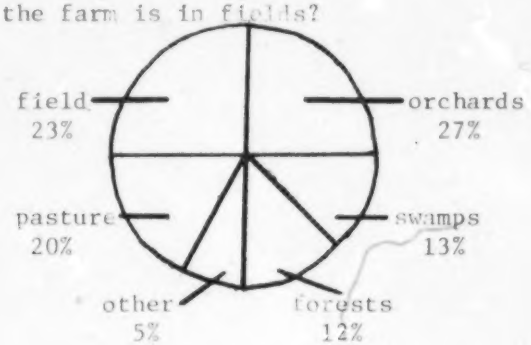
1. What change should Nancy get back? She gave a quarter and 3 dimes to pay for paper that cost 47¢. The tax is 2¢.

   a) 6¢
   b) 8¢
   c) 11¢
   d) 3¢

2. In Pete's room the temperature was 72°, but the cellar temperature was only 47°. What number sentence would be used to find how many degrees colder in the cellar?

   a) 72 + 47 =
   b) 72 - 47 =

3. What percentage of the farm is in fields?



   a) 10%
   b) 12%
   c) 23%
   d) 25%

4. The largest percentage of the farm is used for

   a) orchards
   b) forests
   c) fields
   d) pastures

5. What percentage of the farm is used for growing trees (orchards & forests)?

   a) 39%
   b) 50%
   c) 49%
   d) 15%

6. 14 - 9 =

7. 14 - 2 =

8. 8 1/2
   -2 3/4

9. $36 \div 9 =$

10. Round 0.378 to the nearest tenth.

11. $8 + ____ = 10$

12. Reduce to lowest terms: $2/4 =$

13. What fraction names the part that is shaded?

14. $36 + 10 + 9 =$

15. Find the missing number in the sequence:

    4, 8, 12, ____, 20

Classification for Sample Exercises B.

1) 6, 12, 11

2) 2, 4, 6

3) 3, 10, 1

4) 3, 10, 2

5) 5, 10, 3

6) 4, 1, 6

7) 4, 2, 6

8) 4, 7, 7

9) 4, 1, 9

10) 4, 9, 15

11) 4, 5, 6

12) 4, 7, 1

13) 1, 6, 1

14) 4, 3, 5

15) 6, 3, 4

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